MATH 464 (THEORY OF PROBABILITY)
HOMEWORK 10

FALL 2017

Due on: Tuesday 11-21-2017.

(1) A random variable X has pdf

$$f_X(x) = \begin{cases} e^{1-x} & x \geq 1 \\ 0 & x < 1 \end{cases}$$

Find the moment generating function of X, i.e., $M_X(t)$.

(2) Show that for any random variables X and Y we have

$$\mathrm{Cov}(X+Y, X-Y) = \mathrm{Var}(X) - \mathrm{Var}(Y)$$

(3) Suppose that random variables X and Y are independent. Show that

$$\rho(X+Y, X-Y) = \frac{\mathrm{Var}(X) - \mathrm{Var}(Y)}{\mathrm{Var}(X) + \mathrm{Var}(Y)}$$

(4) Suppose $X_1, \ldots, X_n$ are independent random variables with $\mathbb{E}(X_j) = \mu$ and $\mathrm{Var}(X_j) = \sigma^2$ for all $j = 1, \ldots, n$. Let $S_k = X_1 + \ldots + X_k$. Find $\rho(S_k, S_n)$.